V. L. Rykov
September 08, 2003

Statistical errors for the number of detected particles or hits
(π^0 A_{LL} and many others Analysis Notes in mind)

It is well known that, **for a given integrated luminosity**[1], the number of detected[2] **events**, M, is Poissonian distributed. But, in inclusive measurements[3], the distribution of the number of detected **particles**[4], N, generally is not a Poissonian one. There are two sources for N fluctuations: 1) fluctuations of the particle multiplicity n per event due to physics, limited acceptance, detecting and reconstruction efficiencies, etc; 2) fluctuations of the number of events, M, for a given integrated luminosity. The general formula, which describes N-fluctuations in terms of σ_N (RMS), is as follows: $\sigma_N^2 = \sigma_n^2 \bar{M} + \bar{n}^2 \sigma_M^2$, where $\bar{n}$ and σ_n are for the mean particle multiplicity per event and the width (RMS) of the multiplicity distribution; $\bar{M}$ and σ_M are for the mean number of events for a given integrated luminosity and the RMS of this M distribution. Since the number of events M is Poissonian distributed, then $\sigma_M = \sqrt{\bar{M}} \approx \sqrt{M_{sample}}$, where M_{sample} is the number of actually detected events[5] which can be used and normally is used as an estimate for $\bar{M}$. Then, formula for σ_N transforms to the following: $\sigma_N^2 = \sigma_n^2 \bar{M} + \bar{n}^2 \bar{M} = \left(\sigma_n^2 + \bar{n}^2\right)\bar{M}$, or $\sigma_N = \sqrt{\left(\sigma_n^2 + \bar{n}^2\right)\bar{M}} \approx \sqrt{\left(\sigma_n^2 + \bar{n}^2\right)M_{sample}}$. For $\bar{n}$ and σ_n estimates, the mean and RMS of the measured multiplicity distributions could be taken[6]. The experimental π^0 multiplicity distributions in our measurements are shown in Fig. 14. The relative statistical errors, σ_N / N , in various P_T bins, obtained using the formulae above and data from Fig. 14, are shown in Table 3.

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1 … or exposition time.
2 … and filtered at either trigger, or analysis stages, or both …
3 … or, what is the same, with the inclusive particle and/or hit counting.
4 … or hits of interest, possibly including background due to misidentification, combinatorics, etc.
5 For example, in a particular run.
6 The hit multiplicity distributions become close to Poissonians if the multiplicity fluctuations are **primarily** due to, for example, small detector acceptance or similar. Then, $\sigma_n \approx \sqrt{\bar{n}}$, and $\sigma_N \approx \sqrt{\left(\bar{n} + \bar{n}^2\right)\bar{M}} \approx \sqrt{\bar{n}\left(1+\bar{n}\right)\bar{M}} \approx \sqrt{\left(1+\bar{n}\right)\bar{N}} \approx \sqrt{\left(1+\bar{n}\right)N_{sample}}$. In the limiting case of $\bar{n} \ll 1$, $\sigma_N \approx \sqrt{\bar{N}} \approx \sqrt{N_{sample}}$. It should be underlined that inequality $\bar{n} \ll 1$ **does not** always mean the Poissonian multiplicity distribution and $\sigma_n \approx \sqrt{\bar{n}}$. The example of such a case is using a **relaxed trigger** for the rare event selection.